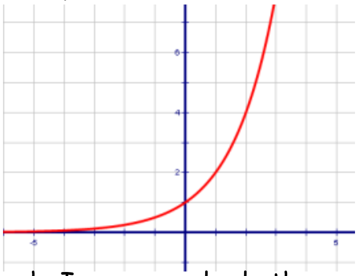
6.6 Exploring the Properties of Exponential Functions

Exponential Growth:



Graph: Increases slowly then quickly

Exponential Decay:

Graph: Decreases quickly then slowly

Asymptote:
A line that a curve will continually approach but never touch

Asymptote:
- x axis
- eq'n: y=0

y will never be zero!

NOTE: for our purposes we will not be looking at a graph that has been translated up or down, that is why the x axis will be the asymptote

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Summary

function:	algebraic model:
Linear - constant 1st differences.	$y = mx + b$
Quadratic - constant 2nd differences.	$y = x^2$
Exponential - Multiplication Pattern of 1st diff.	$y = b^x$ or $y = ca^x$

The following table shows exponential growth:

x	f(x)	
0	1.0000	× 5
1	5.0000	× 5
2	25.0000	× 5
3	125.0000	× 5
4	625.0000	× 5
5	3125.0000	

Growth is exponential if there is a multiplication pattern for consecutive values in the table.
(sometimes easier to look at first differences to see the ratio)

(Recall linear relations have a common first difference and Quadratic relations have a common second difference)

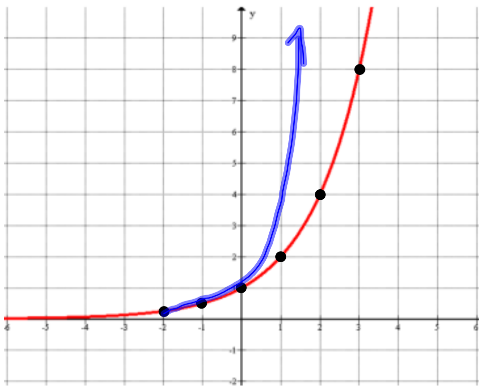
First $f(x) = b^x$ (parent function)

Has the following characteristics:

1. Only exponential if $b > 0$ and $b = 1$; its domain (x) is the set of real numbers, and its range is the set of all positive real numbers
2. If $b > 1$, the greater the value, the faster the growth
3. If $0 < b < 1$, the lesser the value, the faster the decay
4. The function has a horizontal asymptote, which is the x-axis
5. The function has a y-intercept of 1

The graph of $y = 2^x$ is shown to the right

x	y
-3	0.125
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8
4	16



☆ Look at the value of y when x=1
What do you notice?

Tell you the "~~a~~ b" base value

What are the similarities between the graph of $y=2^x$ and $y=3^x$

- Both are exp. growth
- Both have a y int. of 1

What are the differences between the graph of $y=2^x$ and $y=3^x$

The rate at which they increase

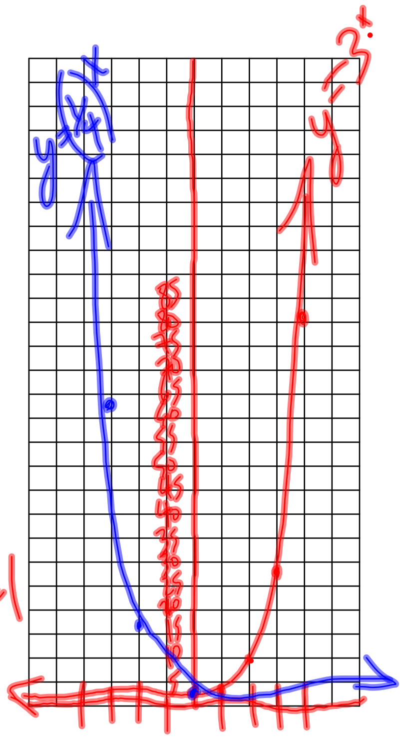
$y = a(b)^x$

Graph of $y = 3^x$

x	y
-3	0.037
-2	0.11
-1	0.33
0	1
1	3
2	9
3	27
4	81

Graph of $y = (1/4)^x$

x	y
-3	64
-2	16
-1	4
0	1
1	0.25
2	0.0625
3	0.016
4	0.0039



What are the x-intercepts for these graphs? none

What are the y-intercepts for these graphs? y intercept = 1

For these graphs, what are the domain and range?

$R = \{y \mid y > 0, y \in R\}$ $X \in R$

What is the equation of the asymptote?

$y = 0$

Match the Graph to the Exponential Equation

(Hint: To find the appropriate base find the value of y when $x = 1$)

$y = 3^x$ D

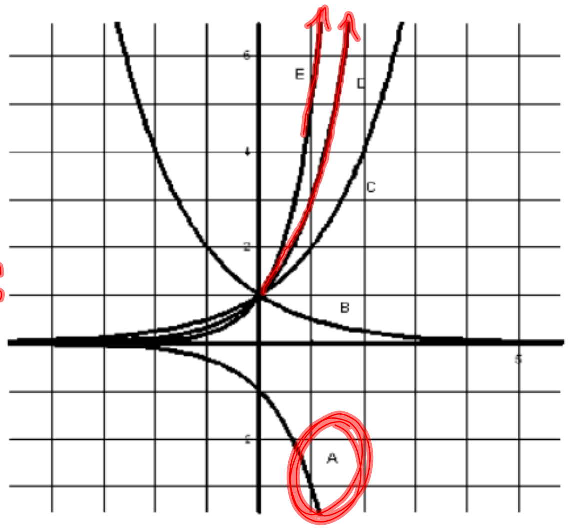
$y = \left(\frac{1}{2}\right)^x$ B

$y = 5^x$ E

$y = 2^x$ C

~~$y = \left(\frac{1}{3}\right)^x$~~ Not on graph

$y = -3^x$ A



$\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^{1}$

Second $f(x) = c(a)^x$

$y = ca^x$

exponent is a variable

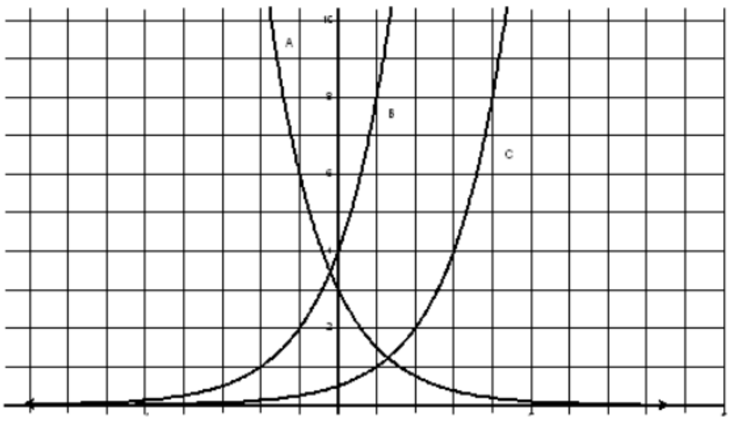
(stretch)

base

- y intercept is 1 if there is no stretch. If c is greater then one then c is the new y intercept and it has a stretch.
- your base is the multiplication pattern of your 1^{st} differences
- from your graph you can see your base by finding your y value at x = 1

  * watch for a stretch*
- Exponential functions need a positive base

2.



$y = 4(2)^x$ ______

$y = \frac{1}{2}(2)^x$ ______

$y = 3\left(\frac{1}{2}\right)^x$ ______

$y = (-6)^x$ ______

Assigned Work: p.423 #1 to 4